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THE DIMENSION OF THE GEARBOX SIGNAL

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1. INTRODUCTION

The power spectrum of the signal measured from a gearbox typically consists of peaks located at integer multiples of the meshing frequency $f_g = Nf_s$, where N is the number of teeth of the gear and f_s is the shaft frequency. In reality, there also exist "modulating" side bands, which have the appearance of narrow-band processes, recording any imperfection stemming from the gears to the entire system set-up [4]. When faults (such as a crack, eccentricity, etc.) develop, these narrow-band components grow, and provide the useful signature to detect any anomaly in the gearbox.

The gear vibration signal can be effectively analyzed by using the synchronous averaging method [1]. It is a novel technique which is able to reject "irrelevant" parts and retain only the essential components in gear meshing dynamics. On top of synchronous averaging, a number of methods are available for crack detection purposes. McFadden suggested using amplitude and phase modulations of the synchronous signal to achieve early crack detection and was successful in a number of cases [10]. Martin and Ismail showed that the change in statistical moments can be used to distinguish between the signals from the good and the cracked gears [9]. In general, a crack size of at least 20% can be detected by the existing method [9]. Recently, much attention has been given to the time–frequency wavelet transform on the synchronous averaged signal [2, 4]. It is a local method and thus has the potential to locate exactly where the cracked tooth is.

In this note, we examine the idea of using the non-linear dynamical system approach for crack detection in a gearbox system [7]. The assumption on which this study is based is that the "modulating" narrow-band process of the gear signal describes some degree of chaos in the gear meshing dynamics. It then implies the potential of using non-linear techniques to detect small changes in the gear vibration due to, e.g., a crack, or "secondary effects" caused by the presence of a crack, such as the imbalance of the shaft, leading to resonance. Our primary interest is to study the sensitivity of using a dimension measure to characterize the vibration of defective gears and to estimate the dimension quantity in real time.

In this study, we find that it is possible to use the dimension approach to detect a 15% crack. The dimension estimate tends to decrease when certain thresholds of the crack size and shaft speed are passed. Since the dimension estimate is a global indicator, it raises concern as to how much of the gear dynamics is captured in the calculation. To address this issue, we studied the recurrence of the embedded gear signal, and sufficient evidence was found to answer this question in the positive. We also computed the filtered signal and found qualitatively similar results. The recurrence technique provides an interesting alternative for the crack detection in a realistic gearbox, since it does not rely on the estimation of a statistical quantity. In the next section, the methods and experimental set-up are described. The results of data analysis and on-line calculations are given in the last section.

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2. METHODS

Using the method of delay, the measurement data $\{x_1, \ldots, x_N\}$ is transformed to a set of vectors $\{y_1, \ldots, y_{N-(m-1)\tau}\}$ in $\mathbf{R}^m$ by

$$y_i \equiv (x_i, x_{i+r}, \ldots x_{i+(m-1)\tau}), \tag{1}$$

where m and τ are called the embedding dimension and delay, respectively [11]. To study $\{y_k\}$, the percentage of time, $\mu(r, y_j)$, for which the trajectory spends in the r-neighborhood of some y_j is estimated. The point-wise dimension can then by defined as

$$d_j = \lim_{r \to 0} \frac{\log \mu(r, y_j)}{\log (r)}, \tag{2}$$

provided that the limit exists. By the theorem of Young and other general results, the property of the point-wise dimension can lead to many subtle consequences of the dynamics [12]. In the experiment, we cannot pursue the rigor of these nice results. Instead, d_j is averaged over a subset of $\{y_k\}$ to give a global indicator d for the signal (see reference [3] for related practical issues). We assume that d is a function of other system parameters, such as the tooth's stiffness, shaft speed and so on. The sensitivity of chaotic dynamics implies that one may be able to use d to capture small variations in these system parameters.

Since d is a global indicator, how much of it is derived from the gear-meshing process is generally not clear. Based on the recurrence property of the gear signal, this issue may be resolved. The "impact" dynamics of the gears imply that it is possible to define the recurrent time T_r to a set of properly chosen r-balls in R^m. T_r is a random variable defined by its density function $P(T_r)$. The density function for the cracked gear should show peaks located at $T_r = 1/f_s$ due to the periodic impacts from the bad tooth. On the other hand, this "regularity" is disrupted in the good gear due to the narrow-band component in the signal. Hence, *a more regular looking $P(T_r)$ indicates potential faults being developed in the gears*.

The experimental set-up of the gearbox and the data acquisition system, are sketched in Figure 1. The test rig consists of a single stage gearbox, two DC electric motors, a DC motor controller and resistors for power dissipation. Commercially available mild

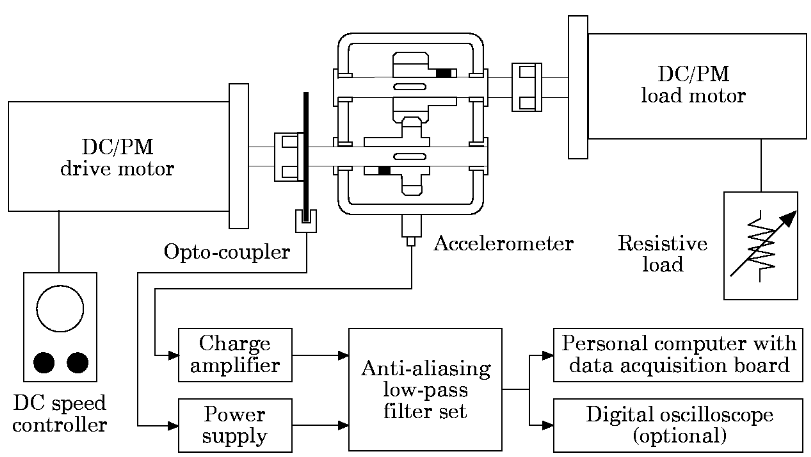

Figure 1. Schematics of the gearbox system.

Figure 2. The 33% crack tooth.

steel gears are used in the experiment. The gears we used have a teeth ratio of 14:16. The crack is deliberately introduced at the driving gear which has 16 teeth, and it is made at the root of the tooth in the direction of the shaft rotation (Figure 2). The vibration signal is measured from the accelerometer attached at the housing of the bearing. The signal is first (analog) low-pass filtered at the cut-off frequency $10f_g$ and then digitized and stored in the computer.

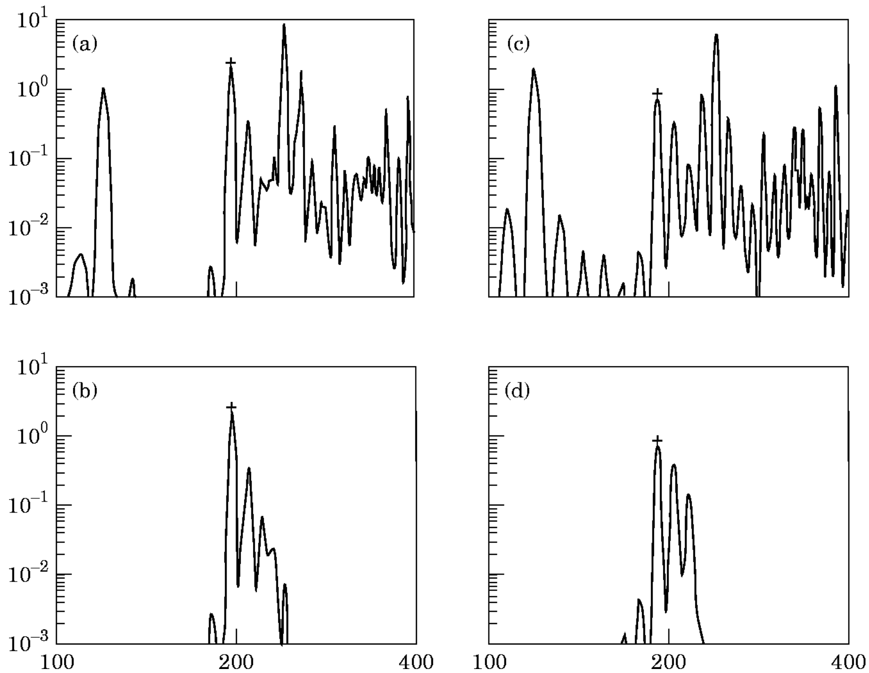

Figure 3. Power spectral density functions at 707 r.p.m for (a) unfiltered 0% crack gear data, (b) filtered 0% crack gear data, (c) unfiltered 15% crack gear data and (d) filtered 15% crack gear data; the + sign denotes the first meshing frequency. The other peaks seen in the figures (at 120 Hz and 240 Hz) result from the distortion of the power line signal (60 Hz) by the DC motor controller (unit of horizontal axis: rad/s).

3. RESULTS

Three different crack sizes, 15%, 25% and 33%, are analyzed in the experiment. Two shaft speeds of 707 r.p.m. ($f_g = 190$ Hz) and 770 r.p.m ($f_g = 207$ Hz) are used for each case. The data is sampled at 5000 Hz and 32 000 points are stored for each case. A typical power spectrum at 707 r.p.m. is shown in Figure 3.

The dimension estimate obtained from equation (2) is shown in Figure 4. The value of d first decreases with the crack size from 0% to 25% and then increases from 25% to 33%. The decrease is the result of the periodic impact from the bad tooth [6]. The increase in d for larger crack size is due to the excitation of other components in the system, which results in the increase of complexity in the dynamics. For the shaft speed of 707 r.p.m., d decreases by 29% for the 15% crack size. Apart from the slight increase from 0% to 15% crack size, the d value calculated at the 770 r.p.m. shaft speed is consistent with that at 707 r.p.m.

The question of how much gear meshing dynamics is captured in the calculation of d can be answered in the positive by the recurrent time analysis. Ideally, the dynamics returns to the r-neighborhood for every gear impact. In reality, the recurrence is random and is characterized by a random recurrent time T_r.

The measured density functions $P(T_r)$ at 707 r.p.m. are shown in Figure 5. It takes less computation time to obtain these results than for the d value. However, more memory space is required, since the ensemble from which $\mu(r, y_i)$ is calculated has to be stored to compute $P(T_r)$. For the cracked gear, peaks located at $T_r = k/f_s$, $k = 1, \ldots$ are seen. They correspond to the periodic impact per revolution from the bad tooth. There appears to be less "structure" in the $P(T_r)$ of the good gear, although peaks located at k/f_s, $k = 1, \ldots$ can still be identified. We believe that this is due to the narrow-band process causing variations in the impact time.

We also study the filtered gear data. The filter is a digital (square) band-pass with the center frequency located at the first meshing frequency f_g. The band is a $\pm 14\%$ fall-off from the first meshing f_g for 707 r.p.m. and $\pm 8\%$ for 770 r.p.m. The dimension estimate

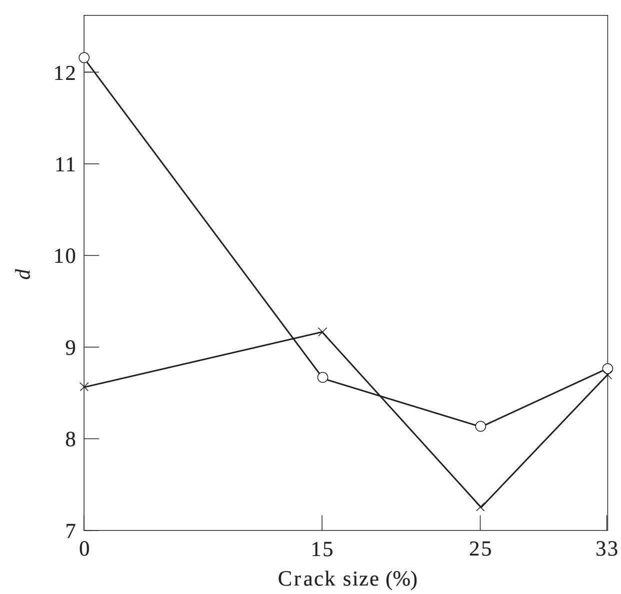

Figure 4. The plot of d versus crack size (unfiltered data, delay = 3, data sampling rate = 5 kHz). ○, 707 r.p.m.; ×, 770 r.p.m.

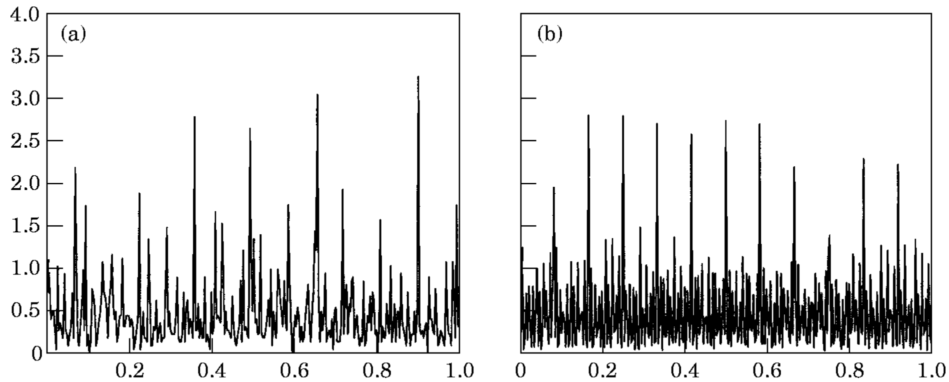

Figure 5. The plot of $P(T_r)$ versus T_r (unfiltered data at 707 r.p.m.): (a) 0% crack; (b) 15% crack.

is naturally smaller (Figure 6). However, a similar trend as seen in the unfiltered case is found. For the 15% crack, the calculated *d*'s show a 10% and a 16% decreases for 707 r.p.m. and 770 r.p.m., respectively. One should note that the 15% difference in the *d* values of the good and bad gears actually results from the very similar-looking spectra (Figures 3(b) and 3(d)). The non-linear approach takes into account the topological property of the time series, which cannot be obtained from amplitude-based calculations. For example, the spectrum of a time series remains unchanged if its phase is randomized. However, this results in a very different topology from the original time series after the delay reconstruction procedure.

These results encourage us to use the dimension estimate *d* for the on-line monitoring of gearbox signals. The on-line approach eliminates the storage problem and saves time for dimension analysis in general. The problem is almost analogous to finding level-crossing statistics in random vibration [5], except that one now has to add a piece of algorithm to do the delay embedding. Once that is accomplished, a memory space

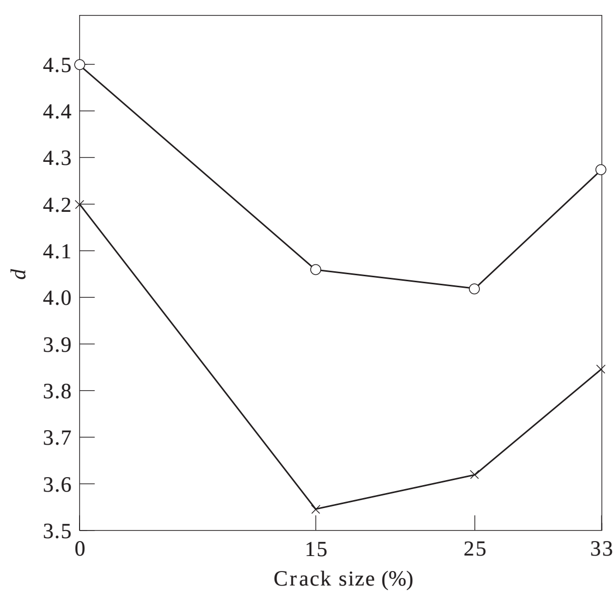

Figure 6. The dimension estimate *d* versus crack size (filtered data, delay = 3, data sampling rate = 5 kHz). ○, 707 r.p.m.; ×, 770 r.p.m.

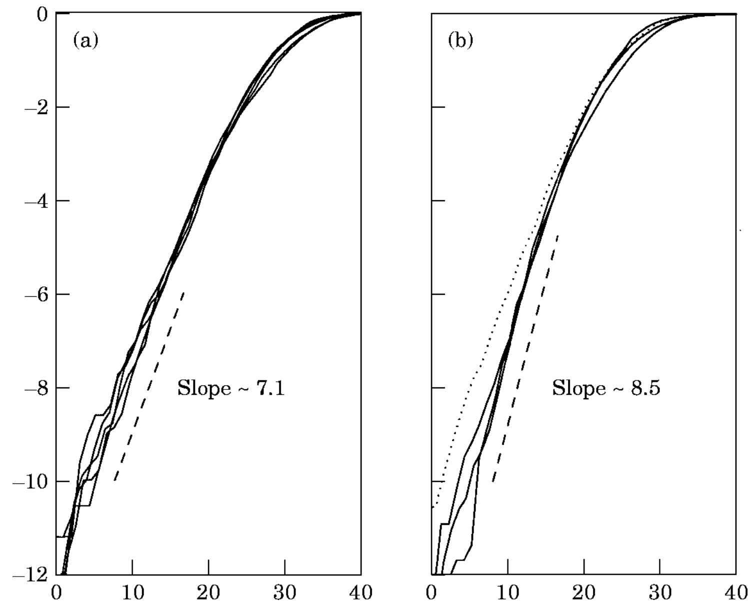

Figure 7. An on-line dimension estimate (horizontal scale is arbitrary). Parameters: 770 r.p.m., unfiltered $r = 3$, sampling frequency = 100 Hz. (a) 25% crack size, $m = 11, 13, \ldots, 19$; (b) 15% crack size, $m = 15, 17, 19$. The dotted line in (b) is given for comparison with the case $m = 17$, 25% crack.

on the order of $2(m-1)r$ elements (see equation (1)) has to be reserved to perform the calculation. A prototype software has been developed and run, on-line, with the gear experiment [8]. The result of the case 770 r.p.m., 25% crack size is shown in Figure 7. The corresponding $\log(\mu)$ versus $\log(r)$ curve and its local slope estimate are given. The slope matches the off-line calculation reasonably well (Figure 3). The speed of the on-line calculation is controlled by the data sampling rate and the m and k parameters of the embedding procedure. For the data reported here, the sampling rate is set at 100 Hz. For a typical run at the speed of 770 r.p.m., a good scaling range can be reached for about 5 minutes ($\sim$30 000 data points).

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